# Joint Decoding: Extracting the Correlation among User Pairs in a Multi-way Relay Channel

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***Abstract*—This paper describes a novel mechanism for joint decoding of the network coded symbols in a multi-way relay node. The mechanism, based on belief propagation algorithm, utilizes the correlation between adjacent network coded symbols to minimize the error propagation problem significantly, compared with previous methods. In case of increasing degree of asynchrony, disjoint decoding exhibits poorer error performance, whereas joint decoding helps to maintain the performance level close to that in the synchronous case both in additive white Gaussian noise and fading channels. Thus, this method adds robustness to the multi-way relay channel against channel imperfections like asynchronism and fading in practical propagation environments.**

***Index Terms*—Asynchronism, belief propagation (BP), error propagation, joint decoding, multi-way relaying.**

## I. Introduction

Relays, capable of providing spatial diversity [1] and extended coverage [2], have become the centre of growing research interest in the arena of cooperative communications. The concept of relaying has evolved through the stages of classical unidirectional relay [1] to bidirectional relay channel [3]–[5] for complete exchange of information between two users. Moreover, efficient schemes like digital and physical layer network coding have been incorporated to improve the performance levels of these bidirectional or two-way relay channels. Digital network coding involves XOR operations on bit streams from the two users, whereas, physical layer network coding utilizes the additive nature of physical electromagnetic waves to further enhance throughput [3]. Physical layer network coding schemes require tight symbol and phase synchronization, which may not always be feasible in practical propagation environments. To deal with such asynchronous system, Lu et al. [6], [7] have designed a framework for decoding in the receiver based on belief propagation method that reduces the penalties of asynchrony.

Recently, as the research interests concentrate more on the multi-user networks than on isolated systems, it is worthwhile to extend the concept of relaying to multi-way relay channel, where multiple users exchange information with the help of a single relay terminal [8], [9]. Gündüz et al. [8] have considered Gaussian multi-way relay channel with decode and forward (DF) strategy, whereas Ong et al. [9] implemented functional decode and forward in binary symmetric channels. In functional binary DF, the relay decodes functions of message pairs, which are simple XOR operations, using time division multiple access [9]. Finally, each user receives the functions from the relay and decodes them sequentially to retrieve all the other users' messages. However, if a user wrongly decodes another user's message, the error propagates through the message extraction process. This problem, termed as error propagation, can affect the system performance adversely both in the cases of DF and amplify and forward (AF) relaying [10], [11].

In the aforementioned research works on multi-way relay channel, the decoding strategy utilized in the relay is a straightforward extension of the decoding process in the two-way relay channel. However, there is a correlation between adjacent network coded symbols received by the relay, that needs to be taken into account to maximize the benefits of a multi-way relay channel. Thus, in the multi-user scenario, the users should be jointly decoded, which is possible through the implementation of belief propagation algorithm. Moreover, to investigate the system performance in a realistic propagation environment, channel imperfections like symbol and phase asynchrony and fading should be considered. Based on the above open questions, we have made the following contributions in this paper that have not been addressed previously, to the best of our knowledge:

1) We design a joint decoding mechanism for the network coded symbols in the multi-way relay node that effectively improves the error propagation problem compared to that of disjoint decoding.
2) We investigate the impact of symbol and phase offsets in a multi-way relay channel and find that the joint decoding mechanism can improve the system performance almost to the level of the synchronous system.
3) We consider Rayleigh fading in synchronous and asynchronous systems and for both joint and disjoint decoding, where joint decoding is found to achieve superior performance.

The rest of the paper is organized as follows. The system model and joint decoding algorithm are described in Section II. In Section III, the algorithm is modified for asynchronous systems. The system performance under fading scenario is discussed in Section IV. Section V provides the numerical simulation results. We conclude the paper in Section VI.

## II. System Model and Proposed Joint Decoding

At first, we consider a binary input Gaussian output synchronous multi-way relay channel with $L$ users and then extend it to include the impacts of asynchronism and fading in Section III and Section IV, respectively. We assume that the users are exchanging their information through a single relay without any direct link between them. The channel model is similar to that of [9]. The complete information exchange among all the users is performed in multiple access phase and broadcast phase. In the first phase, the relay receives the sum of signals from a simultaneously transmitting user pair. In the second phase, the relay broadcasts the decoded messages and all the users receive and store it. Once all the network coded bits have been received, the users retrieve messages from other users through the cancelation of self-information. Thus, an $L-$user relay network would require $L-1$ steps for multiple access phase and then another $L-1$ steps for broadcast phase. For example, in the $\ell^{th}$ step of multiple access phase, only the users, $\ell$ and $\ell+1$ participate in the two-way relaying operation. In the following step, users, $\ell+1$ and $\ell+2$ transmit and so on. Let the $i^{th}$ and $(i+1)^{th}$ user transmit binary messages, $W_i$ and $W_{i+1}$ which are BPSK modulated to $X_i$ and $X_{i+1}$, respectively. The relay receives the signal

$$Y_{i,i+1} = X_i + X_{i+1} + n_1, \tag{1}$$

where $n_1$ is the zero mean additive white Gaussian noise (AWGN) with noise variance $\frac{N_0}{2}$. The relay then performs either disjoint decoding operation for each user pair or can jointly decode the messages of user pairs, as described below.

### A. Disjoint Decoding

The relay decodes the received signal from each user pair using maximum a posterior (MAP) criterion, where the optimum threshold is [3]:

$$\gamma_r = 1 + \frac{N_0}{4}\ln\left(1+\sqrt{1-e^{-\frac{8}{N_0}}}\right). \tag{2}$$

The true network coded symbol transmitted by the sources is given by [9]:

$$V_{i,i+1} = W_i \oplus W_{i+1}. \tag{3}$$

However, in a noisy environment, the symbol detected by the relay is given by $\hat{V}_{i,i+1}$.

### B. Joint Decoding

The relay receives the signal from each user pair and stores it. After receiving all such network coded signals, it performs joint decoding through belief propagation algorithm between network coded bits of different user pairs on a bit-by-bit basis. That is, the belief is passed from the first network coded bit of the first user pair to the first network coded bit of the second user pair up to the $(L-1)^{th}$ user pair and then repeated for all the bits of the packet in the same manner. Belief propagation algorithm computes marginal probability distributions for nodes in a graphical model [12]. The graphical model contains source (or variable) nodes, evidence nodes and compatibility nodes (or constraint nodes or factor nodes) that define the inter-relationship of a group of source nodes [7]. The message passed from a source node involves the product of the incoming messages from nodes connected to it and the message passed from a compatibility node involves the appropriate product of functions corresponding to the source nodes connected to it with a summation operation performed over the product [13]. The belief at any node is proportional to the product of local evidence at that node and all the messages coming to that node [12].

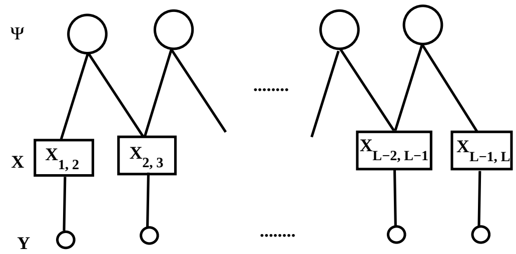



Fig. 1. Tanner graph for joint decoding in a multi-way relay node.

*1) Tanner Graph Formation:* The network coded bits from different user pairs can be applied to construct a Tanner graph, on which belief propagation algorithm can be implemented. The structure of the tanner graph has been shown in Fig. 1. Here, $Y$ denotes the evidence nodes, $\Psi$ denotes the compatibility nodes and $X$ denotes the source nodes. The correlation between two adjacent network coded symbols $X_{a,b}$ and $X_{b,c}$ is represented by:

$$\psi(X_{a,b}, X_{b,c}) = \begin{cases} 1 & \text{if } X_b \text{ in } X_{a,b} \text{ and } X_{b,c} \text{ are equal,} \\ 0 & \text{otherwise.} \end{cases} \tag{4}$$

*2) Message Update:* Messages on each edge of the tanner graph is first updated from left to right and then from right to left. The message from each evidence node is represented by the probability vector, $P = (p^{1,1}, p^{1,-1}, p^{-1,1}, p^{-1,-1})$. Here,

$$p^{x_i,x_{i+1}} = P(X_i = x_i, X_{i+1} = x_{i+1} \mid Y_{i,i+1} = y_{i,i+1}) \tag{5}$$
$$\propto \frac{1}{\sqrt{\pi N_0}} e^{-\frac{(y_{i,i+1}-x_i-x_{i+1})^2}{N_0}},$$

where, $x_i, x_{i+1} \in \{1, -1\}$. The right-bound message from compatibility node to source node is represented by $Q_{C,S} = (q_R^{1,1}, q_R^{1,-1}, q_R^{-1,1}, q_R^{-1,-1})$, where each component $q_R^{x_i,x_{i+1}}$ is defined in (7). Similarly, right-bound message from source node to compatibility node, left-bound message from compatibility node to source node and left-bound message from source node to compatibility node are represented by $R_{S,C} = (r_R^{1,1}, ..., r_R^{-1,-1})$, $R_{C,S} = (r_L^{1,1}, ..., r_L^{-1,-1})$ and $Q_{S,C} = (q_L^{1,1}, ..., q_L^{-1,-1})$, respectively. Here, $r_R^{x_i,x_{i+1}}$ and $r_L^{x_i,x_{i+1}}$ are defined by (6) and (8), respectively and $q_L^{x_i,x_{i+1}}$ has a definition similar to $r_R^{x_i,x_{i+1}}$, as explained in update step 3.

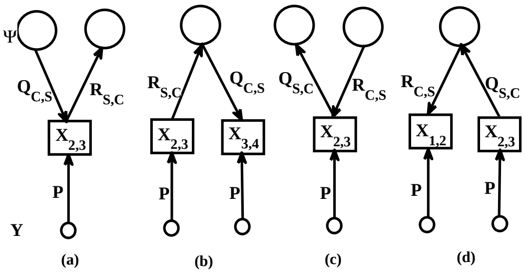



Fig. 2. Message update rules in Tanner graph for joint decoding in a multi-way relay node. (a), (b): From left to right; (c), (d): From right to left.

The update steps are as following:

1) Updating $R_{S,C}$:

$$r_R^{x_i,x_{i+1}} = p^{x_i,x_{i+1}} q_R^{x_i,x_{i+1}}. \quad (6)$$

For the leftmost message, i.e., the network coded bit of the first user pair, the update equation becomes $r_R^{x_i,x_{i+1}} = p^{x_i,x_{i+1}}$.

2) Updating $Q_{C,S}$: If $x_{i+1}$ is the common message between the user pairs involved in the update process, the update equation is given by:

$$q_R^{x_{i+1},1} = q_R^{x_{i+1},-1} = r_R^{1,x_{i+1}} + r_R^{-1,x_{i+1}}. \quad (7)$$

3) Updating $Q_{S,C}$: Similar to step 1 with $r_R^{x_i,x_{i+1}}$ and $q_R^{x_i,x_{i+1}}$ replaced by $q_L^{x_i,x_{i+1}}$ and $r_L^{x_i,x_{i+1}}$, respectively.

4) Updating $R_{C,S}$: If $x_{i+1}$ is the common message between the user pairs involved in the update process, the update equation is given by:

$$r_L^{1,x_{i+1}} = r_L^{-1,x_{i+1}} = q_L^{x_{i+1},1} + q_L^{x_{i+1},-1}. \quad (8)$$

The message update rules have been shown in Fig. 2(a)-(d). The complexity of the update rules are four multiplications (see (6)). Other operations are simple additions. Though this scheme is more complex than the disjoint one, the messages converge after only one iteration. Then the decoding is performed based on the probability tuple:

$$\Big( \sum_{x_i \oplus x_{i+1}=1} p^{x_i,x_{i+1}} q_R^{x_i,x_{i+1}} r_L^{x_i,x_{i+1}}, \sum_{x_i \oplus x_{i+1}=0} p^{x_i,x_{i+1}} q_R^{x_i,x_{i+1}} r_L^{x_i,x_{i+1}} \Big). \quad (9)$$

For the first and last user pair, the tuple is modified by omitting $q_R^{x_i,x_{i+1}}$ and $r_L^{x_i,x_{i+1}}$ from the probability tuple, respectively. The process is repeated for all the bits of the message packet.

After decoding the messages, the relay again modulates them through BPSK and broadcasts to the users. The signals received at the $i^{th}$ and $(i+1)^{th}$ users are denoted by $Y_i^R$ and $Y_{i+1}^R$, respectively, where

$$Y_i^R = Z_{i,i+1} + n_2. \quad (10)$$

Here, $Z_{i,i+1}$ is the BPSK modulated signal from the relay and $n_2$ is AWGN with noise variance $\frac{N_0}{2}$. The users then detect the received signal through MAP criterion, with the optimum threshold given by [10]:

$$\gamma = \frac{N_0}{4} \ln \left( \frac{4}{\text{erfc}\left(\frac{\gamma_r+2}{\sqrt{N_0}}\right) + \text{erfc}\left(\frac{\gamma_r-2}{\sqrt{N_0}}\right) + 2\text{erfc}\left(\frac{\gamma_r}{\sqrt{N_0}}\right)} - 1 \right). \quad (11)$$

The detected symbol is denoted by $\hat{\hat{V}}_{i,i+1}$. When $i^{th}$ user has estimated the network coded information of all such user pairs, it extracts the message of the $(i+1)^{th}$ user by performing XOR operation between its own information bit and the network coded bit and utilizes the extracted information to obtain the information of the $(i+2)^{th}$ user in the same manner. This process is continued until the message of $L^{th}$ user has been decoded. The decoding process is shown in (12).

$$\hat{W}_{i+1} = \hat{\hat{V}}_{i,i+1} \oplus W_i, \quad \hat{W}_{i+2} = \hat{\hat{V}}_{i+1,i+2} \oplus \hat{W}_{i+1}, \quad \cdots, \quad \hat{W}_L = \hat{\hat{V}}_{L-1,L} \oplus \hat{W}_{L-1}. \quad (12)$$

The decoding process is repeated for each user in a total of $2(L-1)$ steps.

## III. Asynchronous Multi-way Relay Channel

In this section, we consider a phase offset $\phi$ and symbol misalignment $\Delta$ between the signals of simultaneously transmitting user pairs, where $0 < \phi < 2\pi$ and $0 < \Delta < 1$. For this case, the relay oversamples the $N$-sample received signal to obtain $2N+1$ signal samples, given by [6]:

$$Y_{i,i+1}[2n-1] = X_i[n] + X_{i+1}[n-1]e^{j\phi} + n_1[2n-1] \quad \text{and} \quad Y_{i,i+1}[2n] = X_i[n] + X_{i+1}[n]e^{j\phi} + n_1[2n]. \quad (13)$$

Here, $1 \le n \le N+1$ with $X_{i+1}[n-1] = 0$ at $n = 1$ and $X_i[n] = 0$ at $n = N+1$. $n_1[2n-1]$ and $n_1[2n]$ are zero-mean complex Gaussian noise with variance $\frac{N_0}{2\Delta}$ and $\frac{N_0}{2(1-\Delta)}$, respectively per dimension.

For disjoint decoding, the tanner graph, similar to that in [6] is constructed for each user pair and the message update equations given in [6] for a two-way relay node can be directly used to decode the network coded messages of each user pair.

For joint decoding, the tanner graph described in section II-B for synchronous case needs to be integrated with that in [6] for asynchronous case. The message passing is performed in two phases. In the first phase, belief propagates from one bit to another within the message packet of each user pair. In the following phase, belief propagates from one user pair to the next one on a bit-by-bit manner. The tanner graph in this case has been shown in Fig. 3. Here, $\Psi'$ denotes the compatibility node for belief propagation within the packet of a certain user pair and $X_{i,i+1}^{m,n}$ corresponds to the oversampled network coded bit based on the $m^{th}$ symbol of the $i^{th}$ user and $n^{th}$ symbol of the $(i+1)^{th}$ user. For odd and even evidence nodes, the probability vectors are given by [6]:

$$p_{x_i,x_{i+1}}^{2n-1} \propto \frac{1}{\sqrt{\pi N_0/\Delta}} e^{-\frac{|y_{i,i+1}[2n-1]-x_i-x_{i+1}e^{j\phi}|^2}{N_0/\Delta}},$$
$$p_{x_i,x_{i+1}}^{2n} \propto \frac{1}{\sqrt{\pi N_0/(1-\Delta)}} e^{-\frac{|y_{i,i+1}[2n]-x_i-x_{i+1}e^{j\phi}|^2}{N_0/(1-\Delta)}}. \quad (14)$$

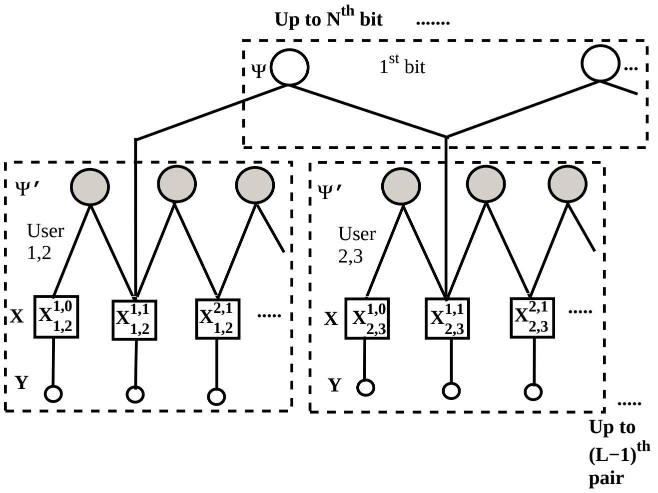



Fig. 3. Tanner graph for joint decoding in asynchronous multi-way relaying.

The message update equations within a certain user pair is similar to that in [6]. These update steps are repeated for all the user pairs and then the message update procedure described in section II-B2 is performed on the messages of different user pairs in a bit-by-bit manner. For the message update in the second phase, the probability vector $p^{x_i,x_{i+1}}$ for a certain bit in (6)-(8) is replaced by the product of right-bound and left-bound messages from compatibility node to source node within a single user pair for that corresponding bit. After this, the relay broadcasts the packets and all the users extract the messages as in the synchronous case.

## IV. Rayleigh Fading in Multi-way Relay Channel

Now we investigate the behavior of both synchronous and asynchronous multi-way relay channel in the presence of Rayleigh fading. We perform this analysis in a time division duplex system where the channels are reciprocal [14]. In a synchronous multi-way relay channel, the received signal is given by:

$$Y_{i,i+1} = h_i X_i + h_{i+1} X_{i+1} + n_1, \tag{15}$$

where $h_i$ and $h_{i+1}$ are the channel coefficients for the $i^{th}$ and $(i+1)^{th}$ user, which are zero mean and unit variance complex valued Gaussian random variables. $n_1$ is the zero mean additive white complex Gaussian noise with noise variance $\frac{N_0}{2}$ per dimension.

For the asynchronous case, the oversampled received signal is given by:

$$Y_{i,i+1}[2n-1] = h_i[n]X_i[n] + h_{i+1}[n-1]X_{i+1}[n-1]e^{j\phi} + n_1[2n-1] \quad \text{and}$$
$$Y_{i,i+1}[2n] = h_i[n]X_i[n] + h_{i+1}[n]X_{i+1}[n]e^{j\phi} + n_1[2n].$$

Then the relay decodes the signal either in a disjoint or joint manner, as described below.

### *A. Disjoint Decoding*

For the synchronous case, the relay decodes the signal using maximum likelihood (ML) criterion, given by [14]. In particular, if

$$e^{-\frac{|Y_{i,i+1}-h_iX_i-h_{i+1}X_{i+1}|^2}{N_0}} + e^{-\frac{|Y_{i,i+1}+h_iX_i+h_{i+1}X_{i+1}|^2}{N_0}} > e^{-\frac{|Y_{i,i+1}-h_iX_i+h_{i+1}X_{i+1}|^2}{N_0}} + e^{-\frac{|Y_{i,i+1}+h_iX_i-h_{i+1}X_{i+1}|^2}{N_0}},$$

then $\hat{V}_{i,i+1} = 1$. In the asynchronous system, the approach is similar to the disjoint decoding in asynchronous AWGN multi-way relay channel described in the previous section, where the probability vector for odd and even evidence nodes are modified in the following manner:

$$p^{2n-1}_{x_i,x_{i+1}} \propto \frac{1}{\sqrt{\pi N_0/\Delta}} e^{-\frac{|y_{i,i+1}[2n-1]-h_i[n]x_i-h_{i+1}[n-1]x_{i+1}e^{j\phi}|^2}{N_0/\Delta}}, \tag{16}$$

$$p^{2n}_{x_i,x_{i+1}} \propto \frac{1}{\sqrt{\pi N_0/(1-\Delta)}} e^{-\frac{|y_{i,i+1}[2n]-h_i[n]x_i-h_{i+1}[n]x_{i+1}e^{j\phi}|^2}{N_0/(1-\Delta)}}.$$

### *B. Joint Decoding*

In the synchronous multi-way fading relay channel, the decoding process is similar to that described in section II-B, with the probability vector modified as:

$$p^{x_i,x_{i+1}} \propto \frac{1}{\sqrt{\pi N_0}} e^{-\frac{|y_{i,i+1}-h_ix_i-h_{i+1}x_{i+1}|^2}{N_0}}. \tag{17}$$

For the asynchronous case, the tanner graph formation and message update procedures are similar to that of joint decoding in asynchronous AWGN multi-way relay channel as described in section III using the probability vectors for evidence nodes from (16).

Once the relay has decoded all the messages, it broadcasts the packets and the signal received at the $i^{th}$ user is:

$$Y_i^R = h_i Z_{i,i+1} + n_2, \tag{18}$$

where $n_2$ is the zero mean additive white complex Gaussian noise with noise variance $\frac{N_0}{2}$ per dimension. The users then detect the received signal through ML criterion [14]:

$$\hat{\hat{V}}_{i,i+1} = \arg\min |Y_i^R - h_i Z_{i,i+1}|^2, \text{where} \quad Z_{i,i+1} \in \{\pm 1\}.$$

Finally, the users perform the message extraction process as in (12).

## V. Numerical Results

This section provides some numerical simulation that exhibits the performance improvement when joint decoding is considered in a multi-way relay channel. In Fig. 4, we can compare the average bit error rate (BER) performance between joint and disjoint decoding in synchronous AWGN multi-way relay channel. For $L = 10$, joint decoding cuts the ratio of average BER of a multi-way relay channel to that of a two-way relay channel from 45 to 20 at a signal to noise ratio (SNR) of 9 dB and improves the SNR level by about 0.5 dB.

Fig. 5 shows the performance of asynchronous AWGN multi-way relay channel for different symbol misalignments while the decoding in the relay is performed either in joint or disjoint manner. The error probabilities in all the cases

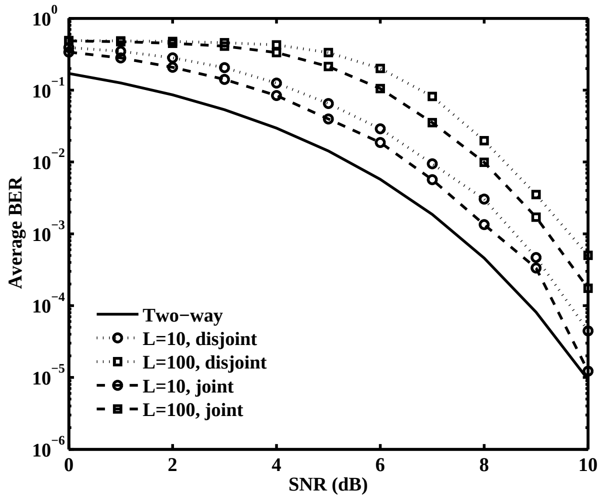



Fig. 4. Average BER of synchronous AWGN multi-way relay channel with joint and disjoint decoding.

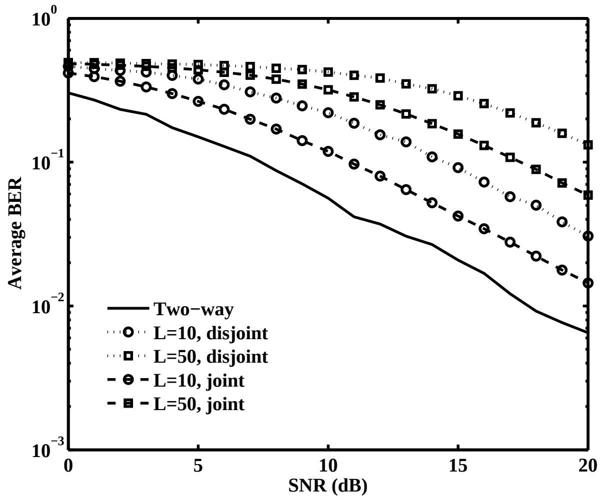



Fig. 6. Average BER of synchronous fading multi-way relay channel with joint and disjoint decoding.

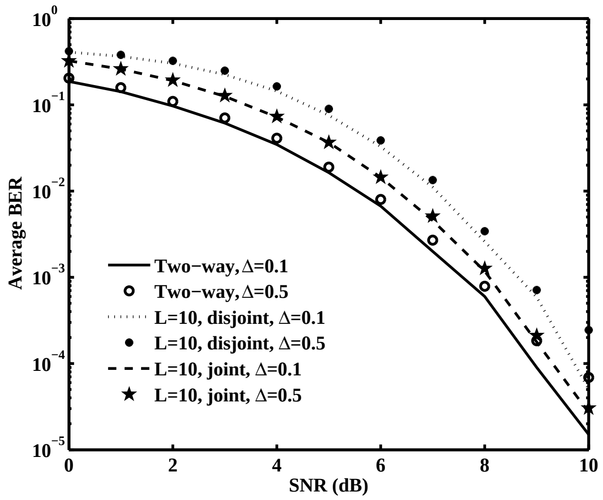



Fig. 5. Average BER of asynchronous AWGN multi-way relay channel with joint and disjoint decoding for different symbol misalignments.

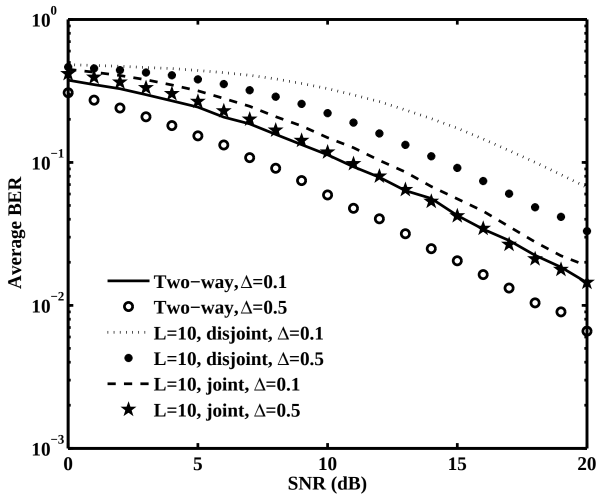



Fig. 7. Average BER of asynchronous fading multi-way relay channel with joint and disjoint decoding for different symbol misalignments.

are averaged over a number of random phase offsets. The ratio of average BER of a multi-way relay channel to that of a two-way relay channel does not change with increasing symbol misalignment when disjoint decoding is performed, which means a larger average BER for a large degree of symbol misalignment. However, for joint decoding, the ratio is smaller for larger degree of symbol misalignment (20 for $\Delta = 0.1$ and 10 for $\Delta = 0.5$ at 9 dB) which indicates that joint decoding helps to keep the average BER to the level of synchronous case even for a large symbol misalignment. Also, joint decoding allows an improvement in SNR by about 1 dB.

Fig. 6 shows the impact of joint decoding in synchronous multi-way relay channel with fading. Similar to the AWGN case, for $L = 10$, joint decoding cuts the ratio of average BER of multi-way relay channel to that of two-way relay channel from 45 to 20 at 18 dB. Moreover, SNR is improved by a larger degree (3.5 dB for 10 users at BER of 0.03 and 4 dB for 50 users at BER of 0.1) than that in AWGN channel.

Joint decoding can also improve the performance of asynchronous multi-way relay channel with fading, which is clearly visible from Fig. 7. An interesting feature in this figure is that 50% symbol misalignment causes a smaller average BER than that caused by 10% misalignment which has been explained in Fig. 9. Because of this feature, for 10% misalignment, improvement in the ratio of average BER of multi-way relay channel to that of two-way relay channel is larger (45 to 10 for Δ=0.1 and 45 to 20 for Δ=0.5 at 18 dB). Results for 90% misalignment are similar to that of 10% and so, they are not shown in this figure.

Fig. 8 shows that the average BER for asynchronous case

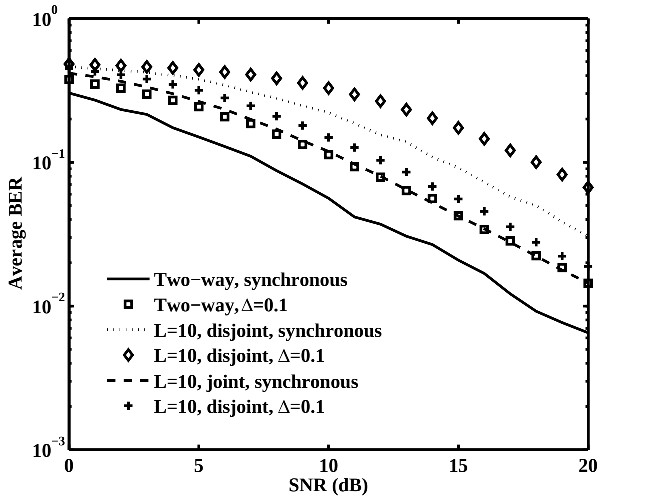
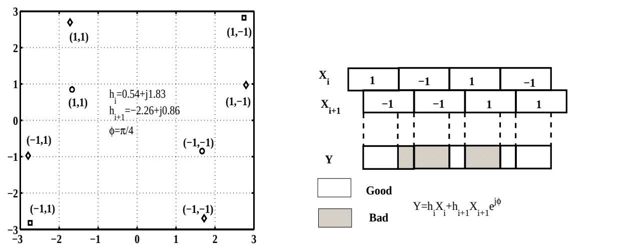



Fig. 8. Average BER comparison of synchronous and asynchronous fading multi-way relay channel with joint and disjoint decoding.




(a) Constellation map (b) Impact of symbol misalignment

Fig. 9. Impact of phase and symbol offset in decoding decision for network coded symbols in a fading channel.

can be improved close to the level of synchronous case when joint decoding is performed. Though this is true for both the AWGN and fading channel, the difference between joint and disjoint decoding is more pronounced in the fading case.

Fig. 9(a) shows an example of constellation map for fading case. The diamonds indicate the constellation for network coded symbols in a fading channel with no phase offset. All of these points are equidistant from the origin. In the presence of phase offset, the constellation is represented by the circles and squares. In this case, the circles are closer to the origin than the squares. It means that there is a higher probability that the received signal will be decoded as '1' than as '0'. However, the actual network coded symbol has equal likelihood to be '1' or '0'. Thus, whenever the decoded received signal is represented by the circles, there is a larger probability that the decision is incorrect and hence these circles can be considered as "bad constellation points" [6]. With similar reasoning, the squares can be referred to as "good constellation points" [6]. When belief propagation is implemented in the relay, the decision about each network coded symbol is influenced by two symbols from the oversampled received signal. Each of these two symbols (odd and even) are assigned a probability vector according to their time duration which depends on the degree of symbol misalignment (see (16)). If these odd and even symbols have unequal durations, there is a chance that in the decoding process, a greater degree of belief is being placed on the bad constellation point rather than the good one, as shown on Fig. 9(b). However, for 50% symbol misalignment, the probabilities are equally weighed and on an average, results into smaller number of wrong decisions than the other cases.

## VI. Conclusion

The joint decoding mechanism, based on belief propagation, is certainly a useful way to lessen the impact of error propagation by utilizing correlation between network coded symbols in a multi-way relay. Also, it can neutralize the influences of symbol and phase offset and fading by maintaining the average BER close to the level of synchronous case, which is not possible through disjoint decoding. Moreover, it provides some gain in the SNR compared to that of disjoint decoding. Our future work will involve the integration of channel coding with this joint decoding mechanism.

## Acknowledgment

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